

Mathalone 2010

Practice Questions

Round 1

Brief Instructions for Round 1

30 questions, 3 minutes/question

- You will be asked 30 mathematical multiple choice questions.
- You will be given ONLY three minutes to answer each question for a total of 90 minutes.
- For multiple choice questions, the correct answer (a, b, c, d or e) must be circled on the answer sheet next to the corresponding question number.
- IF more than one multiple choice answer is correct, then you MUST select all possible answers OTHERWISE your answer will not be counted (since it is incomplete).
- NO CALCULATORS are Allowed.
- The answer sheet will be collected at the end of this round.
- ONLY 10 teams will make it to the next and final round.
- Good Luck 😊

Question 1

The sum of:

$2010 + 201 + 20.1 + 2.01 + 0.201$ is:

a) 2233.311

b) 2244.310

c) 2133.311

d) 1133.310

e) None

Question 2

A dress on sale in a shop is marked at $\$D$. During the discount sale its price is reduced by 15%. Staff are allowed a further 10% reduction on the discounted price. If a staff member buys the dress what will she have to pay in terms of D ?

- a) $0.85D$ b) $0.9D$ c) $0.75D$**
d) $0.765D$ e) None

Question 3

A doctor has two solutions (A & B) that contain different concentrations of a certain medication. A's concentration is 20% while B is 10%. How many cc of A the nurse should mix to obtain 10 cc of 12 % concentration?

- a) 8 b)2 c)5
d)4 e)None

Question 4

A line (D) passing through the point (5, 5) is perpendicular to $5y+6x-24=0$. All of the following points could be on the line (D) except:

- a) (-1, 0) b) (11,10) c) (8,7.5)
d) (2.5,2) e) (-7,-5)

Question 5

Evaluate $|x-y| - x + 2y$ knowing that $x < 0$ and $y > 0$

- a) $-x+2y$ b) y c) $-2x+3y$
d) $-x-y$ e) None

Question 6

Solve the following system of inequalities:

$$\begin{cases} |3x - 9| - 6 < 0 \\ |3x - 2| > x + 1 \end{cases}$$

- a) $1 < x < 5$ b) $x > 3/2$ c) $3/2 < x < 5$
d) $x > 5$ e) None

Question 7

Diana left her house and drove at a constant speed to a conference in another emirate. She picked up Mohd along the way. Two hours after picking up Mohd, they were 140 Kms from Diana's house, and 5 hours after picking up Mohd, they were 344 Kms from Diana's house.

How far (in Kms) from her house was Diana when she picked up Mohd?

- a) 5 b) 10 c)15 d)2 e) None**

Question 8

Mohd can run around a track in 5 minutes while Diana runs around the same track in 9 minutes.

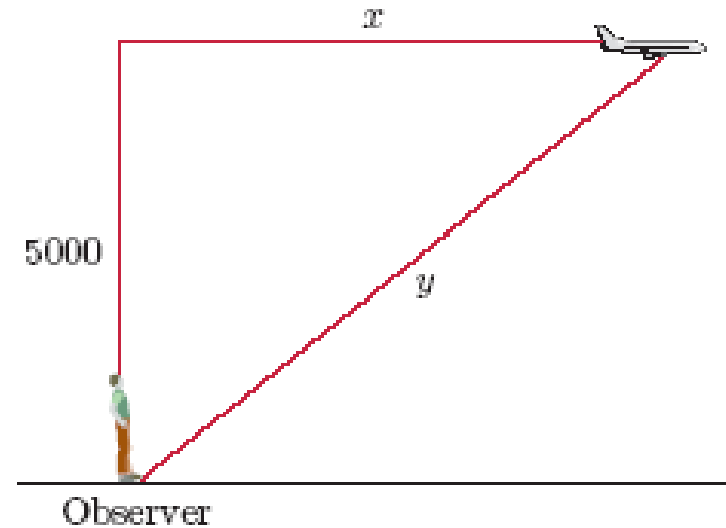
If Mohd and Diana start together, running in the same direction, the number of minutes it will take Mohd to gain one lap on Diana is:

- a) 10 b) 10.25 c) 10.5**
d) 11.25 e) None

Question 9

An airplane flying 390 feet per second at an altitude of 5000 feet flew directly over an observer. The figure below shows the relationship of the airplane to the observer at a later time.

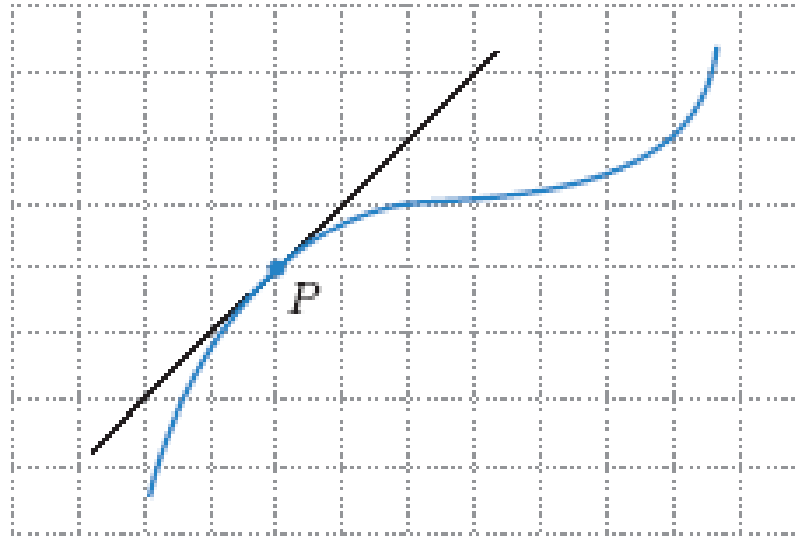
How fast is the distance from the observer to the airplane changing at the time when the airplane is 13,000 feet from the observer?



- a) 360ft/sec
- b) 180 ft/sec
- c) 26000 ft/sec
- d) 12000 ft/sec
- e) None

Question 10

Estimate the slope of the curve at the designated point P .



- a) 1
- b) 2
- c) 0
- d) -1
- e) can not be determined

Question 11

The equation $x^2 + Bx + 2 = 0$ has only one root.

The product of the possible values of B is:

- a) 8 b) -8 c) $2\sqrt{2}$ d) -4 e) None*

Question 12

The width of a rectangle is increasing at a rate of 3 inches per second and its length is increasing at the rate of 4 inches per second. At what rate is the area of the rectangle increasing when its width is 5 inches and its length is 6 inches?

- a) 38 b) 42 c) 39 d) 12 e) None**

Question 13

Find the equation of the tangent line to the curve: $y = \frac{e^x}{x + e^x}$ at $(0, 1)$.

- a) $y+x-1=0$ b) $2y-3x=-1$ c) $y=x+1$
d) $x-2y=-2$ e) None

Question 14

Given the function $f(x) = (\ln x + 1)/x$. Solve for
x: $f'(x) = 0$

- a) $x=1$ b) $x=2$ c) $x=e$
d) $x=0$ e) None

Question 15

On a certain island there are two varieties of people: the *truth tellers*, who always tell the truth and the *liars*, who always lie.

Let the:

Knights: ALWAYS say the truth.

Knaves: ALWAYS lie.

You met two people A and B from the island.

A says: "B is a knight"

B says: "The two of us are of opposite types"

What are A and B?

- a) A is knight, B is Knave
- b) A is knave, B is knight
- c) A is Knave, B is Knave
- d) A is knight, B is Knight
- e) We can not determine using the information given.

Question 16

Given three sets A,B and C. Which of the following statements is always TRUE:

a) $A - (B \cap C) = (A - B) \cup (A - C)$

b) $A - (B - C) = (A - B) - C$

c) $A \cup (B \cap C) \subseteq (A \cup B) \cap C$

d) $(A - C) - (B - C) = A - B$

e) *None of the above*

Question 17

Given the following sequence a_1, a_2, a_3, \dots

1, 0.9, 0.8, 0.7, 0.6,

What is the value of a_{101} ?

- a) 100 b) -9 c) 0.001
d) -9.1 e) None

Question 18

Suppose $U = \{1, 2, \dots, 9\}$,

$A =$ all multiples of 2, $B =$ all multiples of 3, and

$C = \{3, 4, 5, 6, 7\}$.

Find $C - (B - A)$.

- a) $\{3,4,5\}$ b) $\{3\}$ c) $\{4,5,6,7\}$
d) $\{3,6\}$ e) None

Question 19

Given Functions $g : A \rightarrow B$ and $f : B \rightarrow C$ where:

$A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,8,10\}$,

and g and f are defined by

$g = \{(1,b),(2,a),(3,b),(4,a)\}$ and

$f = \{(a,8),(b,10),(c,2)\}$.

Find $f \circ g$.

- a) $\{(1,10),(2,8),(3,10),(4,8)\}$
- b) $\{(10,1),(8,2),(10,3),(8,4)\}$.
- c) $\{(8,b),(10,a),(2,b),(4,a)\}$.
- d) $\{(1,10),(2,8),(3,10)\}$.
- e) None

Question 20

Assume that a, b, c, d , and m are integers with $m > 1$.

Which of the following “theorems” is TRUE.

- a) If $a \equiv b \pmod{m}$, and $a \equiv c \pmod{m}$, then $a \equiv b + c \pmod{m}$.
- b) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv b + d \pmod{m}$.
- c) If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{m}$.
- d) If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{2m}$.
- e) None of the above.

Question 21

Recall that, the factorial of a non-negative integer n , denoted by $n!$, is the product of all positive integers less than or equal to n .

In how many zeros does $30!$ end in ?

- a) Three 0s b) Four 0s c) Five 0s**
d) Seven 0s e) None

Question 22

After being dropped a certain ball always bounces back to $1/2$ of the height of its previous bounce. After the first bounce it reaches a height of 1 meter. How high (in meters) will it reach after its n^{th} bounce?

- a) 2^n b) 2^{-n} c) 2^{1-n} d) 2^{n-1} e) None

Question 23

Suppose that $\log_2(\log_3(\log_5(N))) = 7$.

How many different prime numbers are factors of N ?

- a) 1 b) 2 c) 3 d) 4 e) None**

Question 24

Given that $0 < a < b < c < d$, which of the following is the largest?

a) $\frac{a+b}{c+d}$

b) $\frac{a+d}{b+c}$

c) $\frac{b+c}{a+d}$

d) $\frac{b+d}{a+c}$

e) $\frac{c+d}{a+b}$

Question 25

The expression:

$$\frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$$

Can be written as:

a) $\frac{3 + \sqrt{6} + \sqrt{15}}{6}$

b) $\frac{\sqrt{6} - 2 + \sqrt{10}}{6}$

c) $\frac{\sqrt{6} + 2 + \sqrt{10}}{10}$

d) $\frac{\sqrt{6} + 2 - \sqrt{10}}{6}$

e) *None*

Question 26

If $x/y=3/4$, then which of the following expressions are FALSE:

a) $(x+y)/y=7/4$

b) $y/(y-x)=4$

c) $(x+2y)/x=11/3$

d) $x/2y=3/8$

e) $(x-y)/y=1/4$

Question 27

Let m and n be two positive integers. Consider the following two statements about m and n :

- (I) The number $m - n$ is odd.
- (II) The number $m^2 - n^2$ is odd.

Which of the following statements is true?

- a) (I) is necessary but not sufficient for (II)**
- b) (I) is sufficient but not necessary for (II)**
- c) (I) is neither necessary nor sufficient for (II)**
- d) (I) is necessary and sufficient for (II)**
- e) None of the above**

Question 28

The three positive real numbers x , y , z satisfy the system of two equations:

$$\frac{z}{x+y} = 2 \quad ; \quad \frac{z}{x-y} = 3$$

Which of the following is true:

- a) $x < y < z$ b) $y < z < x$ c) $z < x < y$**
d) $y < x < z$ e) $x < z < y$

Question 29

How many real roots at most can the equation:

$$x/|x| + px + q = 0 \quad \text{have?}$$

- a) 1 b) 2 c) 3 d) 4 e) None**

Question 30

Consider the polynomial $P(x) = x^{2n+1} + x^n - 1$ with $n > 1$.

Which of the following statements is TRUE:

- a) $P(x)$ is a product of two polynomials of even degree
- b) $P(x)$ has integer roots
- c) some of the roots of $P(x)$ are rational numbers
- d) $P(x)$ has a root which is an irrational number
- e) None of these is true

End of Round 1

- **Please remain seated** and put your pens down Immediately.
- Any violators will be **PENALIZED (-2 points)**
- Answer sheets will be collected by the organizers.
- The top 10 teams will be announced after the lunch break.
- Good luck 😊